BELLCOMM, INC.
1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: Velocity Requirements for Direct Launch from KSC to a Circular Polar Orbit 100 Nautical Miles High - Case 720

DATE: December 13, 1967

FROM: H. B. Bosch

ABSTRACT

Idealized velocity requirements are determined for direct ascent from KSC into a polar, circular orbit 100 nautical miles high, by using a dogleg maneuver which avoids flying over populated areas during ascent. Carstens and Edelbaum had previously shown that this method, when optimized with respect to total characteristic velocity, is generally cheaper than a corresponding Hohmann ascent into parking orbit, followed by a pure plane change maneuver.

An idealized analysis (excluding atmospheric drag and gravity loss effects) shows that launching on the southernmost permissible azimuth, with a dogleg maneuver around the eastern tip of Brazil, requires a total velocity of 43,500 fps. For applications to manned missions this is beyond the useful payload capability of the INT-20 vehicle (S-IC/S-IVB). However, if the northernmost permissible azimuth is chosen, with the dogleg maneuver executed around Nova Scotia, the total velocity requirement is 37,300 fps. At this velocity the INT-20 appears capable of injecting a payload of approximately 30,000 pounds.

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MEMORANDUM FOR FILE

Introduction

The most direct way of attaining a polar orbit is to launch on a due north or due south azimuth. However, range safety considerations at KSC do not permit this. In order to avoid flying over populated areas during ascent the launch azimuth must be restricted to be between 44° and 110° from due north. Two idealized launch modes to achieve a circular polar orbit* 100 nautical miles high can be described.

One mode consists of a Hohmann transfer from the launch site to a circular parking orbit 100 nautical miles high. Due to the range safety restrictions this orbit cannot be polar and, therefore, a third impulse must be applied to change the inclination to 90°. In order to minimize the inclination difference between the intermediate and final orbit, this last impulse will be applied at an equator crossing.

In the other mode a direct ballistic ascent is made to intersect the final orbit at some point, where a second impulse simultaneously corrects the flightpath angle, orbital plane and orbital speed (i.e., a dogleg maneuver). The need for a parking orbit is thus eliminated. Note that in this two-impulse mode the launch elevation and range angles are independent variables, as is shown in Appendix 1.

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*The phrase "polar orbit" will be used here without regard to locating the ascending node. Thus, all circular polar orbits of a given altitude are considered equivalent.

Carstens and Edelbaum (reference 1) have shown that, for low altitude orbits, the two-impulse (or direct ascent) mode of attaining a specified orbit is cheaper than the three-impulse mode (using a parking orbit) when the launch elevation and range angles are optimized with respect to total velocity requirement. The intent of this memorandum is to compare the velocity requirements of the two modes for some selected cases. Complete optimization is not appropriate here because, first, we do not select any particular polar orbit and, second, the azimuth or range angle is constrained by range safety considerations. Thus, for each selected case (see table 1) only the launch elevation is optimized.

Numerical Results

For the sake of keeping the analysis simple all trajectories are assumed to be ballistic - i.e., drag and gravity losses are ignored. Also, the earth is considered not to be rotating. This latter assumption would make a difference of 1300 fps or less in the launch velocity. The formulae which were used in the velocity-requirement computations are developed in the two appendixes to this memorandum.

The selected cases are illustrated in figure 1. The first four cases are considered for the northernmost permissible launch on an azimuth of $A_z = 44°$ from due north. The three cases of due east launch ($A_z = 90°$) are for injection into the same polar orbits as in cases 8, 9 and 10, which are on a southeasterly launch with $A_z = 110°$. The launch azimuths and range angles for all these cases are listed in table 1.

The minimum total velocity requirement, ΔV_d, for each of the direct ascent cases of figure 1 was determined by numerically optimizing the value of γ_1 according to the formulae of Appendix 1. The results are plotted versus range angle (ϕ) in figure 2 for the three selected values of launch azimuth. These minimum values are also shown in table 1 as ΔV_d.

The idealized velocity required to launch from a stationary earth to a circular orbit 100 nautical miles high on a Hohmann ellipse is 26,305 fps (see Appendix 2). The impulse required to turn such a circular orbit into a polar one depends on the inclination (i) and is given by formula (2.4) if this impulse occurs at an equator crossing.

The total velocity requirements, ΔV_p, for these three-impulse modes are listed in table 1. They represent the theoretical minimum requirements for launching to a polar orbit by use of a parking orbit and are listed for comparison with the direct ascent mode in each case.

All velocity and payload values given in this memorandum should be taken with the appropriate grain of salt. Such problems as continuous burn and atmospheric drag have been ignored, as was mentioned in the section on Numerical Results. The quoted numbers are therefore intended only to indicate trends and to provide some relative comparisons.

The least expensive of the above examples is case 1 - a northerly launch with a dogleg maneuver in the vicinity of Nova Scotia. At 37,300 fps the INT-20 vehicle is capable of injecting approximately 30,000 pounds* into a polar orbit. The corresponding payload of the Saturn V at that velocity is 82,000 pounds.

For a southerly launch, however, such as the one with the dogleg maneuver around the eastern tip of Brazil (case 8), the Saturn V can deliver only 38,000 pounds at 43,500 fps whereas the corresponding payload for the INT-20 is less than 10,000 pounds.

Discussion

All of the velocity-optimized cases of direct ascent are "lob trajectories": their apogees occur before the target orbit is intercepted. As the point of the dogleg maneuver is approached the velocity vector is pointing downward so that the impact point (in the event of injection motor failure) is only a short distance downrange. If the maneuver has been started but the engine does not burn for the desired length of time, the vehicle will have started to turn in the direction of the final orbit but will not have achieved full orbital speed. In this case the impact point will be farther downrange but along the azimuth on which the vehicle was moving at the instant of cutoff. The range of the ascent trajectory can be shortened somewhat if the resulting locus of possible impact points is considered.

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*Based on figures provided by the Launch Systems Branch of Boeing's Space Division, Huntsville, Alabama, in a study of "Saturn V Intermediate Launch Vehicles".

The total velocity requirement for a given launch azimuth may thereby be reduced. However, figure 2 shows that, in order to have the velocity requirements on an azimuth of 110° competitive with that of case 1, say, the range angle would have to be less than 10°. The impracticality of such a maneuver can be seen from figure 1. Thus, launch on a northeasterly azimuth is still superior to an easterly or southeasterly launch.

Conclusions

The total velocity requirements shown in table 1 and figure 2 show that between 5,500 and 6,700 fps can be saved by making a direct launch (with dogleg maneuver) rather than using a circular parking orbit.

It is also shown that a northeasterly launch (A_z = 44° from due north) requires less total velocity than a southeasterly (A_z = 110°) or an easterly (A_z = 90°) launch, and that it may make the difference between using the INT-20 vehicle or having to use a more powerful one such as the Saturn V.

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Attachments
References
Table 1
Figures 1-4
Appendixes 1, 2

REFERENCES

1. Carstens, J. P., and Edelbaum, T. N.: Optimum Maneuvers for Launching Satellites into Circular Orbits of Arbitrary Radius and Inclination. ARS Journal, July, 1961, pp. 943-949.

2. Wheelon, Albert D.: Free Flight of a Ballistic Missile. ARS Journal, December, 1959, pp. 915-926.

TABLE 1

Case (See Figure 1)	A_z	ϕ	i	Δi	ΔV_p(fps)	ΔV_d(fps)
1	44°	21°	52.4°	55.5°	42,792	37,300
2	44	29	52.4	62.2	42,792	41,000
3	44	37	52.4	70.4	42,792	44,900
4	44	45	52.4	79.8	42,792	49,000
5	90	42	28.5	70.0	52,449	45,900
6	90	49	28.5	67.7	52,449	46,600
7	90	55	28.5	66.0	52,449	47,100
8	110	50	34.3	55.9	50,185	43,500
9	110	57.8	34.3	55.7	50,185	44,500
10	110	65	34.3	55.9	50,185	45,350
11	110	75	34.3	56.9	50,185	46,650

A_z = Launch azimuth measured from due north
ϕ = Range angle (launch to intercept)
i = Inclination of ascent trajectory plane
Δi = Plane change at location of dogleg maneuver
ΔC_p = Idealized total minimum velocity required for parking orbit mode
ΔV_d = Idealized total minimum velocity required for direct ascent mode

APPENDIX 1

The initial velocity (V_1) required to move along an elliptic arc from a radius r_1 to a radius r_2, with a range angle ϕ (see figure 3) is related to the initial flightpath angle (γ_1) by the formula

$$V_1^2 = \frac{\mu}{r_1} \frac{1 - \cos\phi}{(r_1/r_2)\cos^2\gamma_1 - \cos(\phi + \gamma_1)\cos\gamma_1}$$

A derivation of this formula can be found in reference 2*. This can be rewritten in the form

$$V_1^2 = V_c^2 \frac{1 - \cos\phi}{\left(\frac{r_1}{r_2}\cos\gamma_1\right)^2 - \frac{r_1}{r_2}\cos\gamma_1\cos(\phi + \gamma_1)} \tag{1.1}$$

where $V_c = \sqrt{\mu/r_2}$ is the circular velocity at r_2. This formula is valid for any Keplerian trajectory.

The conservation laws for total energy and angular momentum give, respectively,

$$V_2^2 = V_1^2 + 2V_c^2\left(1 - \frac{r_2}{r_1}\right) \tag{1.2}$$

$$r_2 V_2 \cos\gamma_2 = r_1 V_1 \cos\gamma_1 \tag{1.3}$$

Thus, the entire trajectory (in particular, the terminal conditions) will be completely determined once the launch elevation angle (γ_1) has been chosen.

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*Note, however, that the angle γ used in reference 2 is the complement of γ_1.

APPENDIX 1 (cont'd)

Assuming a stationary earth, the first impulse of the ascent trajectory is

$$\Delta V_1 = V_1 \tag{1.4}$$

The second impulse comes when the ascent trajectory intersects the desired circular orbit. From the trigonometry shown in figure 4,

$$\Delta V_2^2 = V_c^2 + V_2^2 - 2V_c\, V_2 \cos\ \gamma_2 \cos \Delta i \tag{1.5}$$

Substituting from (1.2) and (1.3) this becomes

$$\Delta V_2^2 = V_c^2\ (3 - 2\ \frac{r_2}{r_1}) + V_1^2$$

$$- 2\ \frac{r_1}{r_2}\ V_c\ V_1 \cos\ \gamma_1 \cos \Delta i \tag{1.6}$$

The total velocity requirement for the direct ascent mode is

$$\Delta V_d = \Delta V_1 + \Delta V_2 \tag{1.7}$$

The numerical variation of ΔV_d with γ_1 was calculated using r_1 = 3,442 nautical miles, r_2 = 3,542 nautical miles, V_c = 25,567 fps. The latitude of the launch site is 28.5° N.

APPENDIX 2

For the Hohmann transfer we again take

$$\Delta V_1 = V_1 \tag{2.1}$$

To compute the initial velocity we substitute $\phi = 180°$ and $\gamma_1 = 0$ into formula (1.1) to get

$$\Delta V_1 = V_c \frac{r_2}{r_1}\sqrt{\frac{2 r_1}{r_1 + r_2}} \tag{2.2}$$

The second impulse will be the difference between the apogee velocity of the Hohmann ellipse and the circular velocity (V_c) of the parking orbit. Since no plane change takes place here, we substitute $\Delta i = 0$ into formula (1.6) to get

$$\Delta V_2 = V_c \left(1 - \sqrt{\frac{2 r_1}{r_1 + r_2}}\right) \tag{2.3}$$

The only maneuver remaining is polarization of the parking orbit at an equator crossing. This is computed by

$$\Delta V_3 = 2 V_c \sin \left(\frac{90° - i}{2}\right) \tag{2.4}$$

Thus, the total velocity requirement for the parking orbit mode is

$$\Delta V_p = \Delta V_1 + \Delta V_2 + \Delta V_3 \tag{2.5}$$

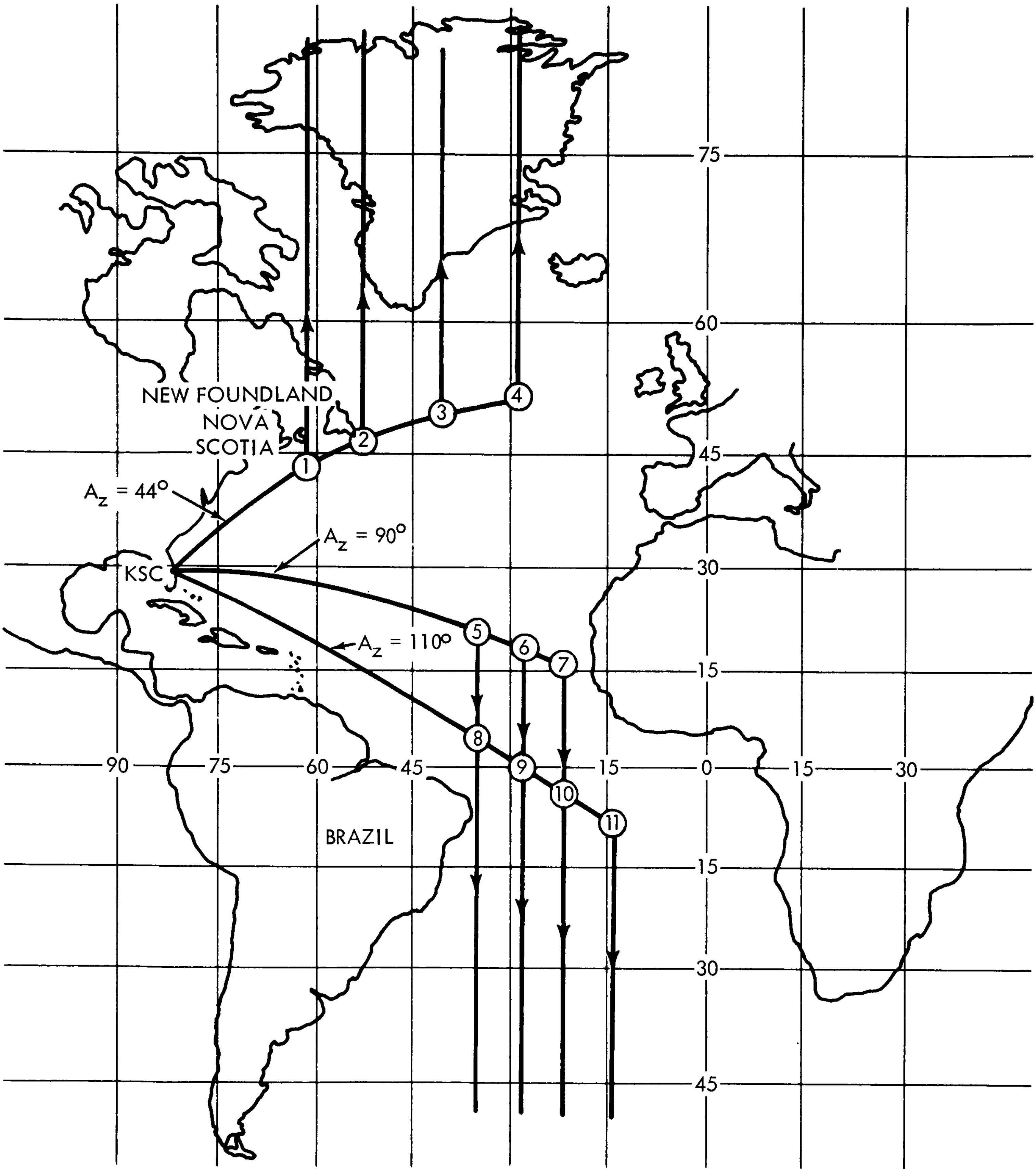

FIGURE 1 - SOME SELECTED CASES OF DIRECT ASCENT INTO POLAR ORBIT USING DOGLEG MANEUVER

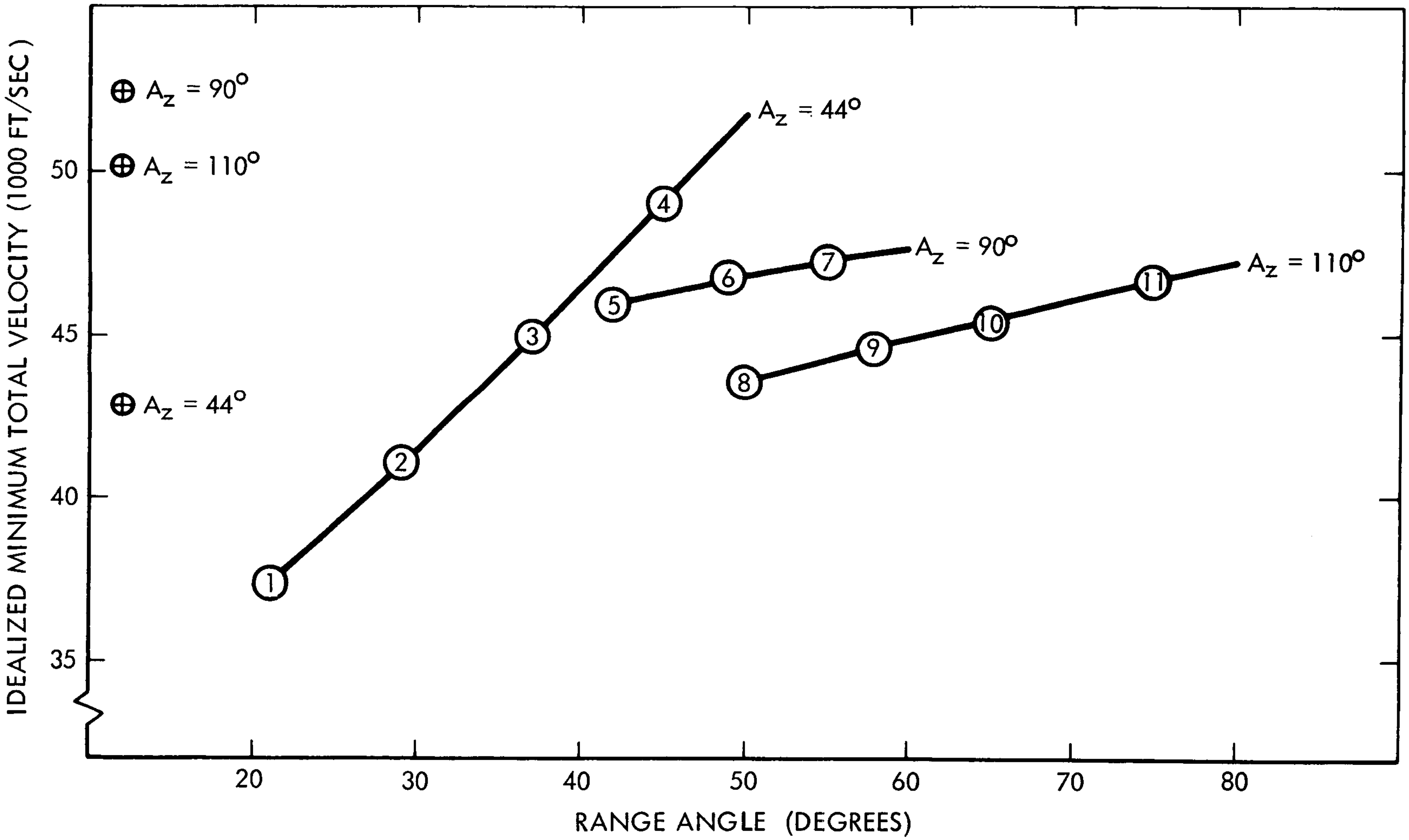

FIGURE 2 - COMPARISON OF MINIMUM TOTAL VELOCITY REQUIREMENTS FOR DIRECT ASCENT MODE (—O—) AND PARKING ORBIT MODE (⊕). NUMBERS IN CIRCLES CORRESPOND TO CASE NUMBERS IN TABLE I.

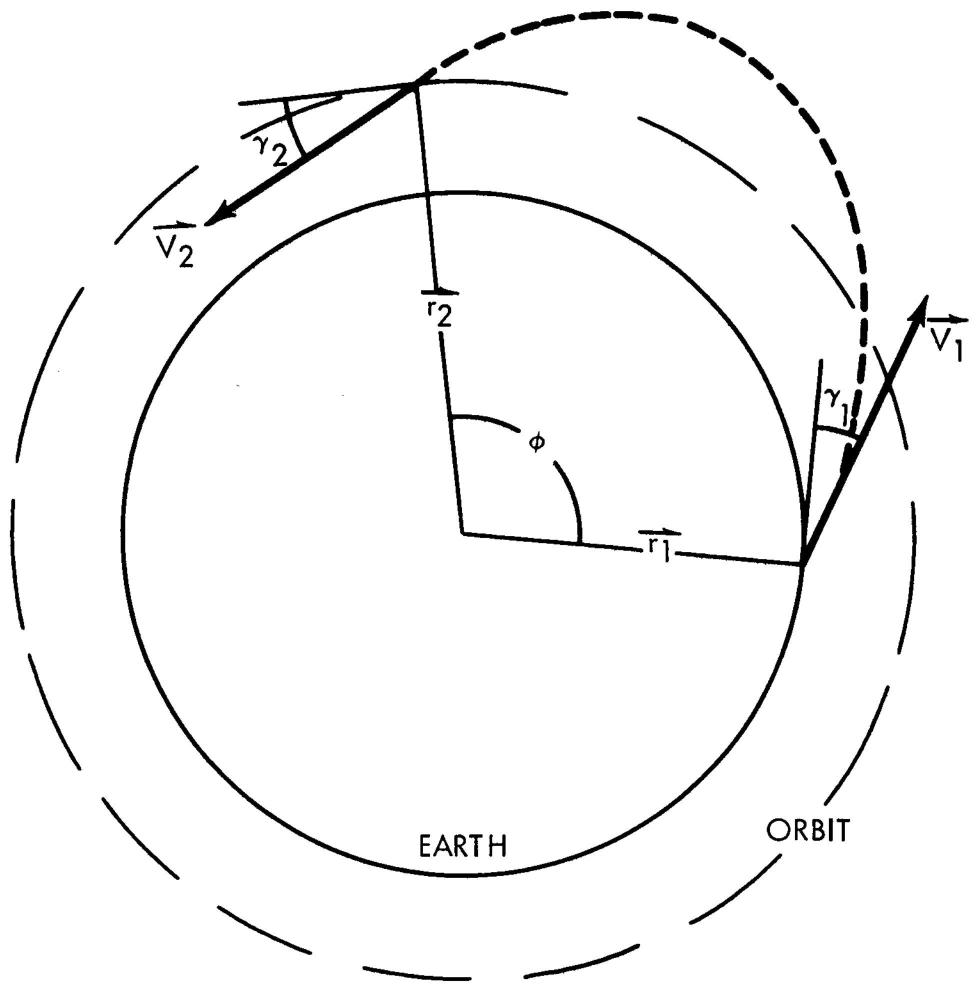

FIGURE 3 - KEPLERIAN PARAMETERS AT LAUNCH AND TERMINAL POINTS OF ASCENT TRAJECTORY.

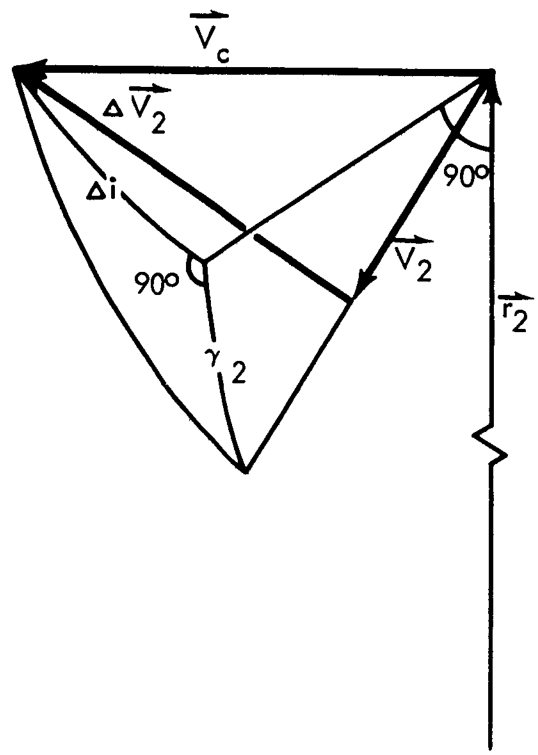

FIGURE 4 - RELATIONSHIP BETWEEN VELOCITY VECTORS AT INTERSECTION OF ASCENT TRAJECTORY $(\vec{V}_2)$ AND CIRCULAR ORBIT $(\vec{V}_c)$.

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